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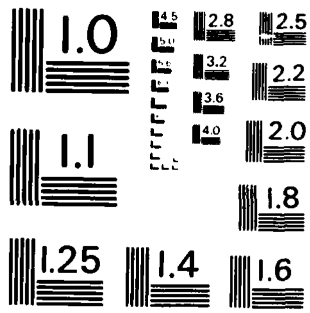

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A unified approach to the theory of correlations in a plasma is presented, based on the BBKGY hierarchy. The theory is applied to a one-component plasma with the Coulomb interaction modified to include effects of the background. Closed integro-differential equations in space and time are obtained for the two-particle correlation function in both the strong and weak coupling limits. In the weak-coupling domain, $\gamma \ll 1$, the time-independent analysis returns the well-known linearized Debye-Huckle result, where γ is the plasma parameter. In the strong-coupling domain with $\gamma \geq 1$, the resulting two-particle 'total' (continued)

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20. · Abstract (continued)

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Unified Theory of Plasma Correlations

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Abstract

A unified approach to the theory of correlations in a plasma is presented, based on the BBKGY hierarchy. The theory is applied to a one-component plasma with the Coulomb interaction modified to include effects of the background. Closed integro-differential equations in space and time are obtained for the two-particle correlation function in both the strong and weak coupling limits. In the weak-coupling domain, $\gamma << 1$, the time-independent analysis returns the well-known linearized Debye-Huckle result, where γ is the plasma parameter. In the strong-coupling domain with $\gamma \gtrsim 1$, the resulting two-particle 'total' correlation function exhibits decaying oscillatory behavior for particle separation of the order of the effective interparticle range.

I. Introduction

Various efforts have undertaken to describe one-component plasmas in two extreme limits. These are the weakly ($\gamma \ll 1$) and strongly ($\gamma \geqslant 1$) coupled domains. In the former limit many such efforts have led to the linearized Debye-Hückle result, using kinetic [1,2,3], statistical mechanical[4,5,6], and numerical[7,8,9] formulations.

In the strong-coupling domain, studies have centered around such problems as plasma turbulence[10,11] and electron correlations at metallic densities[12]. Each approach has applied its own approximation of the correlation functions to the BBKGY hierarchy in order to obtain the dielectric response function[13]. The function thus acquired is then employed in conjunction with the fluctuation-dissipation theorem to obtain a self-consistent solution for the static form factor or, equivalently, the pair correlation function[14].

A more recent study and review of calculations concerning the total internal energy of a one-component plasma in the strong-coupling limit is presented by Gould, et al.[15] Comprehensive expositions on the state of the art of this subject have been given by Kalman and Carini;[16] Baus and Hansen,[8] and Ichimaru.[14]

The relevance of strongly-coupled plasmas to natural phenomena is illustrated in Fig. 1. We note that, in particular, the strong-coupling limit plays an important role in the description of x-ray plasmas, laser fusion devices, and in the interiors of certain super-dense stars.

In the numerical work of Brush, Sahlin and Teller[7], the pair correlation function is evaluated over a large range of values of the plasma paramter, γ. They find that with increasing γ the correlation function passes from a Debye-Hückle form to a decaying, oscillatory form. This numerical study further demonstrates that the effect of the background charge is to alter the effective interparticle interaction away from the bare Coulomb force.

In the present study a unified formulation for the theory of correlations in a one-component plasma is introduced which is valid for weakly and strongly-coupled plasmas. The theory is based on the BBKGY hierarchy and a sequential ordering of the correlation functions. This ordering is specified in terms of the plasma parameter γ. An ε-expansion imbedded within the ordering scheme permits an interactive technique of solution and renders the analysis self-consistent.

To better incorporate the role of the neutralizing background, the effective interparticle force is expanded about the bare Coulomb interaction. This inclusion is motivated by results of previous numerical studies.[7,8] In both the weak and strongly coupled domains, closed space-time integro-differential equations are obtained for the two-particle correlation functions.

In the weak-coupling domain a time-independent analysis returns the well-known linearized Debye-Hückle correlation function.[1-9]

In the strongly-coupled domain a second-order differential equation is obtained for the correlation function. The solution to this equation exhibits a decaying oscillatory structure for particle separations in excess of the order of the effective two-particle interaction range. These purely analytic results are in very good agreement with previous numerical work.[7]

The essential components of this analysis are related as shown in the flow chart given in Fig. 2.

II. Analysis

A. Basic Formulation

We consider an aggregate of charges Ze in a uniform neutralizing background. The s^{th} equation of the BBKGY hierarchy, hereafter called BY_s, is given by (see, for example, Liboff[17])

$$\left[\frac{\partial}{\partial t} + \sum_{i=1}^{s} \underset{\sim}{p}_i \cdot \frac{\partial}{\partial \underset{\sim}{x}_i} + \alpha_o \sum_{i<j}^{s}\sum \underset{\sim}{G}_{ij} \cdot \left(\frac{\partial}{\partial \underset{\sim}{p}_i} - \frac{\partial}{\partial \underset{\sim}{p}_j}\right)\right] F_s$$
$$= - \frac{\alpha_o}{4\pi\gamma_o} \sum_{1}^{s} \frac{\partial}{\partial \underset{\sim}{p}_i} \cdot \int d(s+1)\, \underset{\sim}{G}_{i,s+1} F_{s+1} \tag{1}$$

Parameters are nondimensionalized as follows[18], where barred variables are dimensional.

$$\begin{aligned} &\bar{\underset{\sim}{x}} = r_o \underset{\sim}{x}, \quad \bar{\underset{\sim}{p}} = p_o \underset{\sim}{p} \\ &\bar{\underset{\sim}{G}}_{ij} = \frac{\Phi_o}{r_o} \underset{\sim}{G}_{ij}, \quad \Phi_o = \frac{Z^2 e^2}{r_o} \\ &\bar{t} = \frac{r_o m}{p_o} t \end{aligned} \tag{2}$$

In addition,

$$p_o = m C \quad , \quad m C^2 = k_B T$$

The parameters α_o and γ_o in (1) which emerge as a consequence of nondimensionalization are,

$$\alpha_o = \Phi_o / k_B T \quad , \quad \frac{1}{\gamma_o} = 4\pi n r_o^3 \quad , \quad n = \frac{N}{V} \tag{3}$$

Here Φ_o and r_o are, respectively, the strength and range of the inter-action potential. For the case that $r_o = \lambda_d$, the Debye distance, the coefficients α and γ are written without subscripts.

The nondimensional distribution is then given by[19]

$$F_s = (mC)^{3s} V^s \bar{f}_s \tag{4}$$

where V is the volume of the system and $\bar{f}_s$ is the s-particle joint probability distribution. The function F_s has the normalization,

$$\int F_s \, d1d2\ldots ds = 1 \tag{5}$$

where phase-volume elements are likewise dimensionless. The effective force on the i^{th} ion due to the j^{th} ion is $\bar{\underset{\sim}{G}}_{ij}$.

As noted in the introduction, it has been found in Monte Carlo calculations[7] that the net effect of the background charge is to alter the effective interparticle force away from a simple Coulomb form. Accordingly we write

$$\bar{\underset{\sim}{G}}_{ij} = \hat{\underset{\sim}{x}}_{ij} \frac{\Phi_o}{r_o} \frac{1}{x_{ij}^2} \left[1 - \sum_{n=1}^{\infty} \frac{\mu^n b_n^{\ 2}}{x_{ij}^{\ n-1}} \right] \tag{6}$$

$$\bar{\underset{\sim}{x}}_{ij} = \bar{\underset{\sim}{x}}_i - \bar{\underset{\sim}{x}}_j \tag{6a}$$

The coefficient μ is a parameter of smallness. The coefficients b_n will be addressed later in the paper. Note that the leading correction term in the series (6) has the effect of altering the magnitude but not the form of the Coulomb interaction.

The correlation functions are defined through the Mayer expansions[20]

$$\begin{aligned} F_2(1,2) &= F_1(1)\,F_1(2) + C_2(1,2) \\ F_3(1,2,3) &= F_1(1)\,F_1(2)\,F_1(3) \\ &\quad + F_1(1)\,C_2(2,3) + F_1(2)\,C_2(3,1) + F_1(3)\,C_2(1,2) \\ &\quad + C_3(1,2,3) \\ &\quad\ \vdots \end{aligned} \tag{7}$$

One also writes

$$C_2(1,2) = F_1(1)\, F_1(2)\, h(1,2) \tag{8}$$

where h(1,2) is the so-called 'total correlation function'[21]. In the weak-coupling (or, 'correlationless') domain, one assumes

$$F_1F_1 >> C_2 >> C_3 \ldots \tag{9a}$$

By extension, in the strong-coupling domain we write

$$F_1F_1 << C_2 << C_3 \ldots \tag{9b}$$

These sequential inequalities may both be incorporated into the single set of expansions,

$$\begin{aligned} F_1 &= \gamma^o[F_1^{(o)} + \varepsilon F_1^{(1)} + \ldots] \\ C_2 &= \gamma[C_2^{(o)} + \varepsilon\, C_2^{(1)} + \ldots] \\ C_3 &= \gamma^2[C_3^{(o)} + \varepsilon\, C_3^{(1)} + \ldots] \end{aligned} \tag{10}$$

The ε-factors follow standard perturbation procedure and permit an iterative technique of solution. In (10) we have reintroduced the plasma parameter,

$$\gamma = \frac{1}{4\pi n \lambda_d^3} = \left(\frac{r_o}{\lambda_d}\right)^3 \gamma_o \tag{11}$$

which in this instance serves as a bookkeeping index. With $\gamma << 1$, (10) is seen to return the weak-coupling sequence (9a), whereas with $\gamma \geq 1$, (10) yields the strong-coupling sequence (9b). Thus the expansion (10) serves as a unified description of plasma conditions. The Debye distance λ_d and plasma frequency ω_p which enter (11) are given by

$$\lambda_d{}^2 = \frac{1}{4\pi(eZ)^2}\, \frac{k_BT}{n}\,, \qquad \omega_p{}^2 = \frac{4\pi ne^2Z^2}{m}$$

$$\lambda_d{}^2 \omega_p{}^2 = C^2$$

With (11) we see that in the strong-coupling domain $n^{-1/3} >> \lambda_d$, so that in this limit the Debye distance ceases to represent the range of the two-particle interaction.

It is convenient to introduce the parameter

$$\eta \equiv \left(\frac{\gamma}{\gamma_o}\right)^{1/3} = \frac{r_o}{\lambda_d} = \left(\frac{\alpha_o}{\gamma_o}\right)^{1/2} \tag{12}$$

Consequently the parameter α_o/γ_o that multiplies the interaction integral in (1) may be rewritten as η^2.

Another important plasma parameter found in the literature is

$$\Gamma \equiv (Ze)^2/k_B Ta$$

$$\frac{4\pi}{3} n\, a^3 = 1$$

It is related to γ by

$$\gamma^2 = 3\Gamma^3$$

Using (12) we rewrite (1) in the more concise form

$$(\hat{K}_s + \alpha_o \hat{B}_s) F_s = - \frac{\eta^2}{4\pi} \hat{I}_s F_{s+1} \tag{13}$$

Definition of the operators $\hat{K}_s$, $\hat{B}_s$ and $\hat{I}_s$ follows by comparison with (1). Furthermore we set

$$\hat{B}_s = \sum_{n=o} \mu^n \hat{B}_s^{(n)}$$

$$\hat{I}_s = \sum_{n=o} \mu^n \hat{I}_s^{(n)} \tag{14}$$

in keeping with expansion (6).

In the context of this formalism it is possible to summarize the various possible limiting domains in the form of an α_o-γ diagram[17] as shown in Fig. 3.

B. The Strongly Coupled Domain

In this limit we take $\alpha_o \sim 1$, and the relation $\gamma = \alpha_o \eta$ gives $\eta = \gamma$. Thus BY_1 and BY_2 become, in accord with expansion (7),

$$\hat{K}_1 F_1 = - \frac{\gamma^2}{4\pi} \hat{I}_1 [F_1(1)F_1(2) + C_2] \tag{15a}$$

$$(\hat{K}_2 + \alpha_o \hat{B}_2)[F_1(1)F_1(2) + C_2] =$$
$$- \frac{\gamma^2}{4\pi} \hat{I}_2 [F_1(1)F_1(2)F_1(3) + \sum_{P(ijk)} F_1(i)C_2(j,k)$$
$$+ C_3(1,2,3)] \tag{15b}$$

Here the symbol P(i,j,k) denotes summation over permutations of ijk.

Our chief aim at this point is to obtain a closed kinetic equation for $C_2^{(o)}(1,2)$. To this end a study was made of the ε-dependence of the parameters γ and μ, with $\alpha_o \sim 1$. It was found that $\gamma^2 \sim \varepsilon^{-1/2}$ and $\mu\gamma^2 \sim 1$ are the simplest forms which give closure. (See Appendix A.) Applying the expansions (10) and (14) to (15) with these ε-dependencies of α_o, γ and μ gives the following leading perturbative equations relevant to determining $C_2^{(o)}$.

$$- \frac{\gamma^3}{4\pi} \hat{I}_1^{(o)} C_2^{(o)} = 0 \tag{16a}$$

$$- \frac{\gamma^2}{4\pi} \hat{I}_1^{(o)} F_1(1)F_1(2) = 0 \tag{16b}$$

$$(\hat{K}_2 + \alpha_o \hat{B}_2^{(o)}) C_2^{(o)} = - \frac{\gamma^2}{4\pi} \hat{I}_2^{(1)} \sum_{P(ijk)} F_1(i) C_2^{(o)}(j,k) \tag{17}$$

Here we have written F_1 for $F_1^{(o)}$. We note in passing that (17) is a closed space-time kinetic equation for $C_2^{(o)}(1,2)$. Constraints (16a,b) are consistent with an assumption of spatial homogeneity, so that we may write

$$C_2^{(o)}(i,j) = F_1(\underset{\sim}{p}_i)F_1(\underset{\sim}{p}_j)h^{(o)}(x_{ij}) \tag{18}$$

The right-hand side of (17) is simplified by virtue of (16a) and the vanishing of any space integral over an isotropic vector field. Consequently two terms survive given by

$$\mathrm{RHS}(17) = -\frac{\gamma^2 b_1^2}{4\pi}\{F_1(2)\frac{\partial}{\partial \underset{\sim}{p}_1}F_1(1)\cdot \underset{\sim}{J}_1^{(1,0)} + F_1(1)\frac{\partial}{\partial \underset{\sim}{p}_2}F_1(2)\cdot \underset{\sim}{J}_2^{(1,0)}\}$$

[Go to p. 8]

where

$$b_n{}^2 \underset{\sim}{J}_1{}^{(n,m)} \equiv \int d3\ \underset{\sim}{G}_{13}{}^{(n)} h^{(m)}(x_{23}) \tag{19a}$$

$$b_n{}^2 \underset{\sim}{J}_2{}^{(n,m)} \equiv \int d3\ \underset{\sim}{G}_{23}{}^{(n)} h^{(m)}(x_{13}) \tag{19b}$$

With reference to the results of Appendix B, we may write (dropping the superscript on h)

$$\underset{\sim}{J}_1{}^{(1,0)} = -\ 4\pi \hat{\underset{\sim}{x}}\ x^{-2} \int_0^x d\rho\ \rho^2 h(\rho) \tag{20a}$$

$$\underset{\sim}{J}_2{}^{(1,0)} = -\ \underset{\sim}{J}_1{}^{(1,0)} \tag{20b}$$

Here and henceforth we set $x \equiv x_{12}$.

With (17) and (19) in (16) we obtain

$$\begin{aligned}(\hat{K}_2 + \alpha_o \hat{B}_2{}^{(o)}) F_1(\underset{\sim}{p}_1) F_1(\underset{\sim}{p}_2) h(x) \\ = b_1{}^2 \gamma^2 \left[\frac{\partial}{\partial \underset{\sim}{p}_1} - \frac{\partial}{\partial \underset{\sim}{p}_2}\right] F_1(\underset{\sim}{p}_1) F_1(\underset{\sim}{p}_2) \cdot \hat{\underset{\sim}{x}}\ x^{-2} \int_0^x d\rho\ \rho^2 h(\rho)\end{aligned} \tag{21}$$

In the space-dependent part of $\hat{K}_2$ we use the transformation

$$\underset{\sim}{x}_1 + \underset{\sim}{x}_2 \equiv \underset{\sim}{X} \qquad \underset{\sim}{x}_1 - \underset{\sim}{x}_2 \equiv \underset{\sim}{x}$$

in which case

$$\hat{K}_2 h(x) = \hat{\underset{\sim}{x}} \cdot (\underset{\sim}{p}_1 - \underset{\sim}{p}_2) \frac{\partial}{\partial x} h(x) \tag{22}$$

where we have neglected the time derivative in $\hat{K}_2$. With the repulsive bare Coulomb interaction, it follows from (6) and (14) that

$$\hat{B}_2{}^{(o)} = \hat{\underset{\sim}{x}} \cdot x^{-2} \left[\frac{\partial}{\partial \underset{\sim}{p}_1} - \frac{\partial}{\partial \underset{\sim}{p}_2}\right] \tag{23}$$

Employing the preceding two expressions in (21) gives

$$\hat{\underset{\sim}{x}} \cdot (\underset{\sim}{p}_1 - \underset{\sim}{p}_2) F_1 F_1 \frac{\partial}{\partial x} h(x) + \alpha_o \hat{\underset{\sim}{x}} \cdot x^{-2} \left[\frac{\partial}{\partial \underset{\sim}{p}_1} - \frac{\partial}{\partial \underset{\sim}{p}_2} \right] F_1(p_1) F_1(p_2) h$$

$$= b_1{}^2 \gamma^2 \left[\frac{\partial}{\partial \underset{\sim}{p}_1} - \frac{\partial}{\partial \underset{\sim}{p}_2} \right] F_1(p_1) F_1(p_2) \cdot \hat{\underset{\sim}{x}}\, x^{-2} \int_o^x d\rho\, \rho^2 h(\rho) \tag{24}$$

Assuming finally that F_1 is isotropic in momentum space, we may write

$$\frac{\partial}{\partial \underset{\sim}{p}} F_1(p) = \hat{\underset{\sim}{p}} \frac{\partial}{\partial p} F_1(p) \tag{25}$$

and (24) becomes

$$\hat{\underset{\sim}{x}} \cdot \underset{\sim}{p}_1 \left[F_1 F_1 \frac{\partial}{\partial x} h(x) + \alpha_o x^{-2} \frac{F_1(p_2)}{p_1} h(x) \frac{\partial}{\partial p_1} F_1(p_1) \right.$$

$$\left. - b_1{}^2 \gamma^2 \frac{F_1(p_2)}{p_1} \frac{\partial}{\partial p_1} F_1(p_1) x^{-2} \int_o^x d\rho\, \rho^2 h(\rho) \right] \tag{26}$$

$$= \hat{\underset{\sim}{x}} \cdot \underset{\sim}{p}_2 \left[F_1 F_1 \frac{\partial}{\partial x} h(x) + \alpha_o x^{-2} \frac{F_1(p_1)}{p_2} h(x) \frac{\partial}{\partial p_2} F_1(p_2) \right.$$

$$\left. - b_1{}^2 \gamma^2 \frac{F_1(p_1)}{p_2} \frac{\partial}{\partial p_2} F_1(p_2) x^{-2} \int_o^x d\rho\, \rho^2 h(\rho) \right]$$

Operating on this equation with $\int d\underset{\sim}{p}_2$ eliminates the RHS, and since $\hat{\underset{\sim}{x}} \cdot \underset{\sim}{p}_1$ is generally nonvanishing, there remains

$$F_1(p_1) \frac{\partial}{\partial x} h(x) + \frac{\alpha_o}{x^2} h(x) \frac{1}{p_1} \frac{\partial}{\partial p_1} F_1(p_1)$$

$$- b_1{}^2 \gamma^2 \frac{1}{p_1} \frac{\partial}{\partial p_1} F_1(p_1)\, x^{-2} \int_o^x d\rho\, \rho^2 h(\rho) = 0 \tag{27}$$

Integration over $\underset{\sim}{p}_1$ gives

$$\frac{\partial}{\partial x} h(x) - \alpha_o Y^2 x^{-2} h(x) + b_1{}^2 \gamma^2 Y^2 x^{-2} \int_o^x d\rho\, \rho^2 h(\rho) = 0 \tag{28}$$

where

$$Y^2 \equiv -\int dp_1 \frac{1}{p_1} \frac{\partial}{\partial p_1} F_1(p_1) \tag{29}$$

and may be assumed positive.

Differentiating (28) with respect to x gives

$$h''(x) + \frac{2\alpha_o Y^2}{x^3} h(x) - \alpha_o Y^2 x^{-2} h'(x)$$

$$+ b_1{}^2 \gamma^2 Y^2 \left[h(x) - \frac{2}{x^3} \int_o^x d\rho \, \rho^2 h(\rho) \right] = 0 \tag{30}$$

Using (28) to solve for the integral term in (30) yields the differential equation

$$h'' + \left[2y^{-1} - \kappa y^{-2} \right] h' + h = 0 \tag{31}$$

where

$$y \equiv kx \qquad k = b_1 Y \gamma \qquad \kappa \equiv \alpha_o Y^2 k \tag{31a}$$

Employing the transformation [23]

$$h(y) = \frac{1}{y} e^{-\sigma/y} u(y) \tag{32}$$

$$\sigma \equiv \kappa/2$$

in (31) gives

$$u'' + (1 - \frac{\sigma^2}{y^4}) u = 0 \tag{33}$$

This equation is Schroedinger-like in form and we may conclude[24] that for $y^2 > \sigma$, $h(y)$ is oscillatory, whereas for $y^2 < \sigma$, $h(y)$ is exponential. The explicit form of the solutions in these domains may be obtained in two extremes:

<u>$y^2 >> \sigma$</u>

We are left with

$$u'' + u = 0$$

which has the solution

$$u = A \cos y + B \sin y$$

With (32) we obtain

$$h^{>}(y) = \frac{e^{-\kappa/2y}}{y}\left[A \sin y + B \cos y\right] \tag{35}$$

$\underline{y^2 << \sigma}$

In this case (33) reduces to

$$u'' - \frac{\sigma^2}{y^4} u = 0 \tag{36}$$

which has the solution[24]

$$u(y) = y(\bar{A}\, e^{\sigma/y} + \bar{B}\, e^{-\sigma/y})$$

With (32), this gives

$$h^{<}(y) = \bar{B}\, e^{-\kappa/y} \tag{37}$$

where we have set $\bar{A} = 0$ because (31) does not allow a constant solution. As previously noted, the oscillatory behavior (35) of $h(y)$ in the domain $y^2 >> \sigma$, is in very good agreement with previous numerical studies[7,8].

We now estimate the value of the leading coefficient b_1 in (6). This is accomplished through comparison of the wave number K (defined below) of $h^{>}(y)$ with the numerical value k_{MC} obtained in Monte Carlo studies[7,8]. These results imply that in the vicinity of $\gamma \simeq 50$ (i.e., $\Gamma \simeq 10$)

$$k_{MC} \simeq \frac{2\pi}{\beta a} = \xi/\lambda_d \gamma^{1/3} \quad ; \quad a^3 = 3/4\pi n$$

$$\beta \simeq 1.5 \quad , \quad \xi^2 \simeq 8.4$$

The wavenumber K is given by

$$y = K\bar{x}$$

$$K = \frac{k}{r_0} = \frac{b_1 \gamma \gamma}{r_0} \tag{38}$$

From (11), $\lambda_d = r_0/\gamma$. Furthermore, assuming a Maxwellian form for

$F_1(p_1)$ gives $Y = 1$. With these values

$$K = b_1/\lambda_d$$

and comparison with k_{MC} yields

$$b_1 = \xi\gamma^{-1/3} = \kappa_o 3^{-0.17}\Gamma^{-1/2}$$

which with the values cited above gives $b_1 = 0.8$. With this result at hand, we may estimate the value of interparticle separation at which oscillation of $h(\bar{x})$ ensues. Such oscillation was found from (33) to occur for

$$y^2 \geq \kappa/2$$

or, equivalently

$$\left(\frac{\bar{x}}{r_o}\right)^2 \geq \frac{1}{2b_1\gamma}$$

Inserting our previous finding

$$b_1\gamma = \xi\gamma^{2/3} \approx \gamma^{2/3},$$

gives

$$\bar{x} \geq r_o/\gamma^{1/3}$$

Thus oscillation of $h(\bar{x})$ may be expected for interparticle displacement greater than or equal to the range of the two-particle interaction. The initiation of such oscillation may be interpreted as the onset of phase transition in the plasma[7,32]. Furthermore the wavenumber K of these oscillations (38), is seen to grow with γ. This behavior may be associated with the fluid-solid phase transition[33] at large γ, found in numerical studies.[7,34]

C. The Weak-Coupling Case

This situation has been studied by many individuals[1-9] and as we have noted previously it gives the linearized Debye-Hückle result. In the kinetic domain this limit yields the Vlasov equation, or more generally, the Balescu-Lenard Equation.[26-33]

An essential element of these studies is that the correlation functions are assumed to be perturbatively small. Accordingly we set $\gamma \sim \varepsilon$ in (10). Insofar as $r_o = \lambda_d$ in this limit, it follows that $\eta^2 = \alpha/\gamma = 1$, which fixes $\alpha \sim \varepsilon$. We note that this limit is equivalent to the classical Rosenbluth-Rostoker ansatz.[31] With these constraints and $\mu \sim \varepsilon$, (6) gives the following leading equations for $C_2{}^{(o)}$ as derived from (13)

$$\hat{K}_1 F_1{}^{(o)} = - \frac{\eta^2}{4\pi} \hat{I}_1{}^{(o)} F_1{}^{(o)} F_1{}^{(o)} \tag{39a}$$

$$\hat{K}_1 F_1{}^{(1)}(1) = - \frac{\eta^2}{4\pi} \hat{I}_1{}^{(o)} [C_2{}^{(o)}(1,2) + F_1{}^{(o)}(1) F_1{}^{(1)}(2) + F_1{}^{(1)}(1) F_1{}^{(o)}(2)] \tag{39b}$$

$$\hat{K}_2 [C_2{}^{(o)}(1,2) + F_1{}^{(o)}(1) F_1{}^{(1)}(2) + F_1{}^{(1)}(1) F_1{}^{(o)}(2)] + \alpha \hat{B}_2{}^{(o)} F_1{}^{(o)}(1) F_1{}^{(o)}(2) =$$
$$- \frac{\eta^2}{4\pi} \hat{I}_2{}^{(o)} \left[\sum_{P(ijk)} F_1{}^{(o)}(i) C_2{}^{(o)}(jk) + \sum_{P(ijk)} F^{(o)}(i) F^{(o)}(j) F^{(1)}(k) \right] \tag{39c}$$

We must solve (39) and (40) simultaneously for $C_2{}^{(o)}(1,2)$. First note that (39a) may be rewritten in two equivalent ways:

$$F_1{}^{(1)}(2) \hat{K}_1(1) F_1{}^{(o)}(1) = - \frac{\eta^2}{4\pi} F_1{}^{(1)}(2) \hat{I}_1{}^{(o)}(1) F_1{}^{(o)}(1) F_1{}^{(o)}(3)$$

$$F_1{}^{(1)}(1) \hat{K}_1(2) F_1{}^{(o)}(2) = - \frac{\eta^2}{4\pi} F_1{}^{(1)}(1) \hat{I}_1{}^{(o)}(2) F_1{}^{(o)}(2) F_1{}^{(o)}(3)$$

Similarly (39b) gives

$$F_1^{(o)}(2)\hat{K}_1(1)F_1^{(1)}(1) = -\frac{\eta^2}{4\pi} F_1^{(o)}(2)\hat{I}_1^{(o)}(1)[C_2^{(o)}(1,3) + F_1^{(o)}(1)F_1^{(1)}(3) + F_1^{(1)}(1)F_1^{(o)}(3)]$$

$$F_1^{(o)}(1)\hat{K}_1(2)F_1^{(1)}(2) = -\frac{\eta^2}{4\pi} F_1^{(o)}(1)\hat{I}_1^{(o)}(2)[C_2^{(o)}(2,3) + F_1^{(o)}(2)F_1^{(1)}(3) + F_1^{(1)}(2)F_1^{(o)}(3)]$$

Adding these four equations and subtracting the result from (39c) yields

$$\hat{K}_2C_2^{(o)}(1,2)+\alpha\hat{B}_2^{(o)}F_1^{(o)}F_1^{(o)} = -\frac{\eta^2}{4\pi}\{\hat{I}_2^{(o)}\ P(i\bar{j}k)F_1^{(o)}(i)C_2^{(o)}(jk) - F_1^{(o)}(2)\hat{I}_1^{(o)}(1)C_2^{(o)}(1,3)-F_1^{(o)}(1)\hat{I}_1^{(o)}(2)C_2^{(o)}(2,3)\}$$

Four terms remain on the RHS of this equation. Of these, two vanish by integration of the isotropic vector field $\underset{\sim}{G}_{ij}$, leaving

$$\hat{K}_2C_2^{(o)}(1,2) + \alpha B_2^{(o)}F_1^{(o)}F_1^{(o)} = -\frac{\eta^2}{4\pi}\{\hat{I}_1^{(o)}(1)F_1^{(o)}(1)C_2^{(o)}(2,3) + \hat{I}_1(2)F_1^{(o)}(2)C_2^{(o)}(1,3)\} \qquad (40)$$

Here we have obtained (as in the strongly-coupled analysis) a closed space-time-dependent equation of motion for $C_2^{(o)}$. The RHS of this equation simplifies to

$$\mathrm{RHS}(40) = -\frac{\eta^2}{4\pi}\{F_1^{(o)}(2)\frac{\partial}{\partial \underset{\sim}{p}_1}F_1^{(o)}(1)\cdot\underset{\sim}{J}_1^{(0,0)} + F_1^{(o)}(1)\frac{\partial}{\partial \underset{\sim}{p}_2}F_1^{(o)}(2)\cdot\underset{\sim}{J}_2^{(0,0)}\} \qquad (41)$$

where the $\underset{\sim}{J}$ integrals are defined in (19), and again we find that

$$\underset{\sim}{J}_1^{(0,0)} = 4\pi\hat{\underset{\sim}{x}}\ x^{-2}\int_0^x d\rho\ \rho^2 h(\rho) \tag{42a}$$

$$\underset{\sim}{J}_1^{(0,0)} = -\ \underset{\sim}{J}_2^{(0,0)} \tag{42b}$$

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Substituting (18) and (42) into (40) we obtain

$$\hat{K}_2 F_1 F_1 h(x) + \alpha \hat{B}_2^{(o)} F_1 F_1 =$$

$$- \eta^2 \left[\frac{\partial}{\partial \underset{\sim}{p}_1} - \frac{\partial}{\partial \underset{\sim}{p}_2} \right] F_1 F_1 \cdot \hat{\underset{\sim}{x}}\, x^{-2} \int_o^x d\rho\, \rho^2 h(\rho) \qquad (43)$$

Making use of (22) and (23) in the LHS of (43) gives

$$\hat{\underset{\sim}{x}} \cdot (\underset{\sim}{p}_1 - \underset{\sim}{p}_2) F_1 F_1 h + \alpha \hat{\underset{\sim}{x}} \cdot x^{-2} \left(\frac{\partial}{\partial \underset{\sim}{p}_1} - \frac{\partial}{\partial \underset{\sim}{p}_2} \right) F_1 F_1 =$$

$$- \eta^2 \left[\frac{\partial}{\partial \underset{\sim}{p}_1} - \frac{\partial}{\partial \underset{\sim}{p}_2} \right] F_1 F_1 \cdot \hat{\underset{\sim}{x}}\, x^{-2} \int_o^x d\rho\, \rho^2 h(\rho) \qquad (44)$$

Furthermore, with (25) the preceding equation becomes

$$\hat{\underset{\sim}{x}} \cdot \underset{\sim}{p}_1 \left[F_1 F_1 \frac{\partial}{\partial x} h + \frac{\alpha}{x^2} \frac{F_1(p_2)}{p_1} \frac{\partial}{\partial p_1} F_1(p_1) + \eta^2 \frac{F_1(p_2)}{p_1} \frac{\partial}{\partial p_1} F_1(p_1) x^{-2} \int_o^x d\rho\, \rho^2 h(\rho) \right] \qquad (45)$$

$$= \hat{\underset{\sim}{x}} \cdot \underset{\sim}{p}_2 \left[F_1 F_1 \frac{\partial}{\partial x} h + \frac{\alpha}{x^2} \frac{F_1(p_1)}{p_2} \frac{\partial}{\partial p_2} F_1(p_2) + \eta^2 \frac{F_1(p_1)}{p_2} \frac{\partial}{\partial p_2} F_1(p_2) x^{-2} \int_o^x d\rho\, \rho^2 h(\rho) \right]$$

Repeating the procedure leading to (27), we find that (45) yields

$$F_1(p_1) \frac{\partial}{\partial x} h + \frac{\alpha}{x^2} \frac{1}{p_1} \frac{\partial}{\partial p_1} F_1(p_1) + \eta^2 \frac{1}{p_1} \frac{\partial}{\partial p_1} F_1(p_1) x^{-2} \int_o^x d\rho\, \rho^2 h(\rho) = 0 \qquad (46)$$

Integration over $\underset{\sim}{p}_1$ gives

$$\frac{\partial}{\partial x} h - \frac{\alpha}{x^2} Y^2 - \eta^2 Y^2 x^{-2} \int_o^x d\rho\, \rho^2 h(\rho) = 0 \qquad (47)$$

Differentiating (47) leads to

$$h'' + \frac{2\alpha}{x^3} Y^2 + 2\eta^2 Y^2 x^{-3} \int_0^x d\rho\, \rho^2 h(\rho) - \eta^2 Y^2 h = 0 \tag{48}$$

Combining the last two equations gives

$$h'' + \frac{2h'}{y} - h = 0 \tag{49}$$

where

$$y = \eta Y x \tag{49a}$$

The general solution to (48) may be written

$$h(y) = D_1 \frac{e^{-y}}{y} + D_2 \frac{e^{y}}{y} \tag{50}$$

We need to affirm that (49) is a solution to the parent integral equation (47), which in terms of y is

$$\frac{d}{dy} h - \frac{1}{y^2} \left[\alpha Y^3 \eta + \int_0^y dy\, y^2 h(y) \right] = 0 \tag{51}$$

Inserting (49) into (50) leads to the constraint

$$D_1 + D_2 = - \alpha Y^3 \eta \tag{52}$$

Setting $D_2 = 0$ for physical reasons leaves the solution

$$h(y) = - \frac{\alpha Y^3 \eta}{y} e^{-y} \tag{53}$$

For a Maxwellian, Y = 1. Furthermore, in the present limit we recall that $\eta = 1$, so that y = x, and (52) may be written

$$h(x) = - \alpha \frac{e^{-x}}{x} \tag{54}$$

This is the well-known linearized Debye-Hückle result[35], which follows to first-order in α from an expansion of the more precise nonlinear Debye-Hückle form

$$h(x) = e^{-\phi_d(x)} - 1 \tag{55}$$

where

$$\phi_d(x) = \alpha\frac{e^{-x}}{x} \tag{55a}$$

Whereas the Debye-Hückle form (54) agrees with the linearized result (53) for $x >> 1$, we see that at the origin $h(x)$ diverges, but h_d has the correct value $h_d(o) = -1$.

III. Conclusions

We have described a unified formulation of correlations in a one-component plasma, based on the plasma parameter γ, which is valid in both the weakly and strongly-coupled domains. The theory is based on the BBKGY hierarchy together with expansions of the two-particle interaction and correlation functions.

In the weak-coupling domain, the analysis returns the familiar linearized Debye-Hückle correlation function. In the strong-coupling domain a second-order differential equation for the total correlation function is found which yields an exponentially damped solution near the origin. For particle separation on the order of the range of the two-particle interaction, the equation gives a Bessel-like oscillatory solution. These features of the correlation function are in excellent agreement with previous numerical studies.

In both domains, closed space-time dependent integro-differential equations were obtained for the two-particle correlation functions. We anticipate that future study will find important applications of these results both in plasma and condensed matter physics.

Appendix A

Here we examine the decoupling condition on γ employed in Section B. With expansions (6) and (10) in BY_2 (15b), we write

$$\left[\hat{K}_2 + \alpha_0 \sum_{t=0}^{\infty} \mu^t \hat{B}_2^{(t)}\right]\left[F_1F_1 + \gamma \sum_{n=0}^{\infty} \varepsilon^n C_2^{(n)}\right]$$

$$= -\frac{\gamma^2}{4\pi}\left[\sum_{r=0}^{\infty} \mu^r \hat{I}_2^{(r)}\right]\left[F_1F_1F_1 + \gamma \sum_{n=0}^{\infty} \varepsilon^n C_2^{(n)} + \gamma^2 \sum_{m=0}^{\infty} \varepsilon^m C_3^{(m)}\right] \tag{A1}$$

The F_1 functions are expanded as in (10). We seek a kinetic equation for $C_2^{(0)}$ consistent with $\alpha_0 = 0(1)$. In general, such an equation includes $\hat{K}$, $\hat{B}$ and $\hat{I}$ terms. The mere requirement of a $\hat{K}$ term indicates that the equation sought must be $0(\gamma)$, which leads to the condition

$$\gamma = \mu^t \gamma = \mu^r \gamma^3 \tag{A2}$$

The leftmost equality gives $t = 0$, with $\mu \neq 1$. This specifies inclusion of the $\hat{B}_2^{(0)}$ term in our equation. The rightmost equality gives

$$\mu^r \gamma^2 = 1$$

The simplest choice which includes effects of the background is $r = 1$ which specifies inclusion of the $\hat{I}_2^{(1)}$ term and

$$\gamma^2 \mu = 1 \tag{A3}$$

Finally, decoupling the $I_2^{(1)} C_2^{(0)}$ term from all other $C^{(n)}$ terms [in the RHS of (A1)] requires that

$$\mu\gamma \neq \gamma \mu^q \varepsilon^n \qquad q \geq 0\ , \quad n \geq 1 \tag{A4}$$

$$\mu\gamma \neq \gamma^2 \mu^s \varepsilon^m \qquad s \geq 0\ , \quad m \geq 0 \tag{A5}$$

These relations give, respectively,

$$\mu^{q-1}\varepsilon^{n} \neq 1 \qquad q \geqslant 0\ , \quad n \geqslant 1 \tag{A6}$$

$$\gamma\mu^{s-1}\varepsilon^{m} \neq 1 \qquad s \geqslant 0\ , \quad m \geqslant 0 \tag{A7}$$

Using (A3) in (A6) yields

$$\gamma \neq \varepsilon^{n/2(q-1)} \qquad q \geqslant 0\ , \quad n \geqslant 1 \tag{A8}$$

The strong-coupling condition ($\gamma \gtrsim 1$) requires

$$\varepsilon^{n/2(q-1)} \gtrsim 1\ , \quad q \geqslant 0\ , \quad n \geqslant 1 \tag{A9}$$

This allows as the only possibility, $q = 0$. Therefore, (A8) requires

$$\gamma \neq \varepsilon^{-n/2} \qquad n \geqslant 1 \tag{A10}$$

Similarly, with (A3) in (A7) it follows that

$$\varepsilon^{m/2s-3} \gtrsim 1\ , \quad s \geqslant 0\ , \quad m \geqslant 0 \tag{A11}$$

This leads to the cases $s = 0,1$ and thus

$$\gamma \neq \varepsilon^{-m/3}\ , \quad \varepsilon^{-m} \qquad m \geqslant 0 \tag{A12}$$

Combining results (A10) and (A12), the final conditions on γ are summarized by

$$\gamma = \varepsilon^{-a/b} \qquad a \geqslant 1\ , \quad b \geqslant 4\ , \quad \frac{a}{b} \neq m\ , \quad \frac{m}{2}\ , \quad \frac{m}{3} \tag{A13}$$

where $m \geqslant 0$.

Any one of these allowable orders of γ leads to the sought-after decoupled equation for $C_2^{(0)}$ in the strong-coupling limit. In this analysis we select the simplest case: $a = 1$, $b = 4$, corresponding to

$$\gamma^2 = \varepsilon^{-1/2} \tag{A14}$$

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Appendix B

Consider the integral (19a). We set

$$\underset{\sim}{\rho} \equiv \underset{\sim}{x}_{23} \equiv \underset{\sim}{x}_2 - \underset{\sim}{x}_3 \tag{B1}$$

$$\underset{\sim}{x}_{13} = \underset{\sim}{\rho} + \underset{\sim}{x}$$

in which case for n = 1, m = 0, we find

$$\underset{\sim}{J}_1^{(1,0)} = \frac{\partial}{\partial \underset{\sim}{x}} \int d\underset{\sim}{\rho}\, \frac{h(\rho)}{|\underset{\sim}{\rho} + \underset{\sim}{x}|} \tag{B2}$$

In spherical coordinates with

$$d\underset{\sim}{\rho} \equiv \rho^2 \sin\Theta\, d\Theta\, d\phi\, d\rho$$

$$\underset{\sim}{\rho} \cdot \underset{\sim}{x} = x\rho \cos\Theta \tag{B3}$$

$$|\underset{\sim}{\rho} + \underset{\sim}{x}| = (\rho^2 + 2\underset{\sim}{x}\cdot\underset{\sim}{\rho} + x^2)^{\frac{1}{2}}$$

and x held fixed along the z-axis throughout the integration we obtain

$$\underset{\sim}{J}_1^{(1,0)} = 2\pi \frac{\partial}{\partial \underset{\sim}{x}} \int_0^\infty d\rho\, \rho^2 h(\rho) \int_0^\pi d\Theta \sin\Theta\, (\rho^2 + 2x\rho\cos\Theta + x^2)^{-\frac{1}{2}}$$

Performing the Θ-integration gives

$$\underset{\sim}{J}_1^{(1,0)} = -2\pi \frac{\partial}{\partial \underset{\sim}{x}} \int_0^\infty d\rho\, \rho^2 h(\rho) \left\{ \frac{1}{x\rho} \left[\rho^2 + 2x\rho\cos\Theta + x^2\right]^{\frac{1}{2}} \right\} \Bigg|_0^\pi$$

or, equivalently,

$$\underset{\sim}{J}_1^{(1,0)} = -2\pi \frac{\partial}{\partial \underset{\sim}{x}} \int_0^\infty d\rho\, \frac{\rho}{x} h(\rho) \left[(\rho^2 - 2x\rho + x^2)^{\frac{1}{2}} - (\rho^2 + 2x\rho + x^2)^{\frac{1}{2}}\right]$$

$$= -2\pi \frac{\partial}{\partial \underset{\sim}{x}} \int_0^\infty d\rho\, \frac{\rho}{x} h(\rho) \left[|\rho - x| - |\rho + x|\right]$$

$$= 4\pi \frac{\partial}{\partial \underset{\sim}{x}} \left[\int_0^x d\rho\, \frac{\rho^2}{x} h(\rho) + \int_x^\infty d\rho\, \rho h(\rho)\right]$$

Differentiating, one finds

$$\underset{\sim}{J}_1^{(1,0)} = - 4\pi \, \hat{\underset{\sim}{x}} \, x^{-2} \int_0^x d\rho \, \rho^2 h(\rho) \tag{B4}$$

in agreement with (20a).

For the integral (19b) we set

$$\underset{\sim}{\rho} \equiv \underset{\sim}{x}_{13} \equiv \underset{\sim}{x}_1 - \underset{\sim}{x}_3 \tag{B5}$$

$$\underset{\sim}{x}_{23} = \underset{\sim}{\rho} - \underset{\sim}{x}$$

which leads to

$$\underset{\sim}{J}_2^{(1,0)} = - \frac{\partial}{\partial \underset{\sim}{x}} \int d\rho \, \frac{h(\rho)}{|\underset{\sim}{\rho} - \underset{\sim}{x}|} \tag{B6}$$

Repeating the preceding calculations gives

$$\underset{\sim}{J}_2^{(1,0)} = - \underset{\sim}{J}_1^{(1,0)} \tag{B7}$$

We turn next to the integrals in (41), in which case n = 0, m = 0. With (B1) we find

$$\underset{\sim}{J}_1^{(0,0)} = - \frac{\partial}{\partial \underset{\sim}{x}} \int d\underset{\sim}{\rho} \, \frac{h(\rho)}{|\underset{\sim}{\rho} + \underset{\sim}{x}|} \tag{B8}$$

and with (B5) we obtain

$$\underset{\sim}{J}_2^{(0,0)} = \frac{\partial}{\partial \underset{\sim}{x}} \int d\underset{\sim}{\rho} \, \frac{h(\rho)}{|\underset{\sim}{\rho} - \underset{\sim}{x}|} \tag{B9}$$

Following the analysis above gives

$$\underset{\sim}{J}_1^{(0,0)} = 4\pi \, \hat{\underset{\sim}{x}} \, x^{-2} \int_0^x d\rho \, \rho^2 h(\rho)$$

$$\underset{\sim}{J}_2^{(0,0)} = - \underset{\sim}{J}_1^{(0,0)}$$

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Figure Captions

Figure 1

The plasma parameter γ for n and T (with Z = 1) showing weakly and strongly coupled (shaded) domains.

Figure 2

Flow chart explaining the analysis.

Figure 3

α_o-γ plot showing weakly coupled (WCP) and strongly coupled (SCP) plasma domains.

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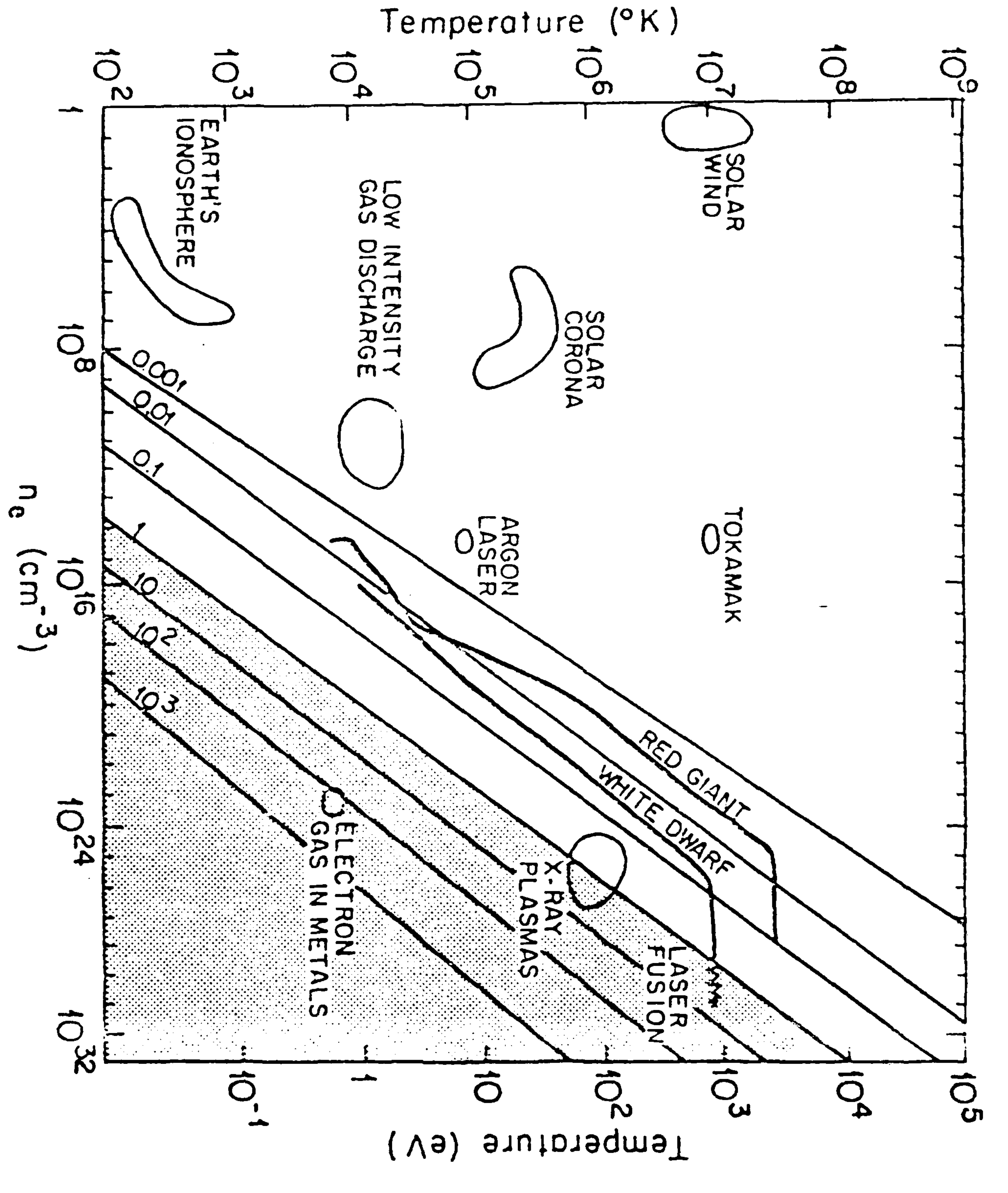


Values of the plasma parameter γ as a function of electron number density and temperature.

FIGURE 1

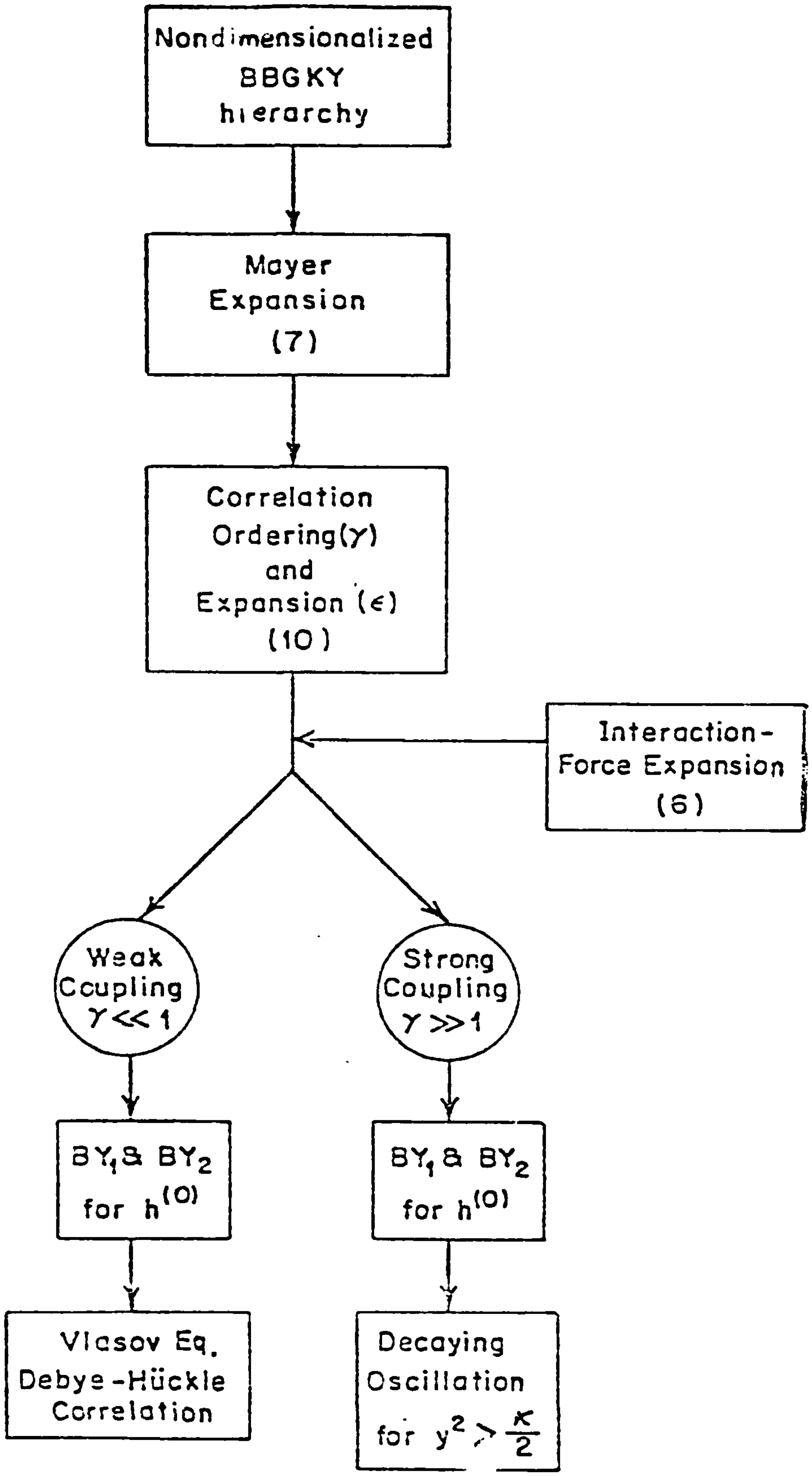
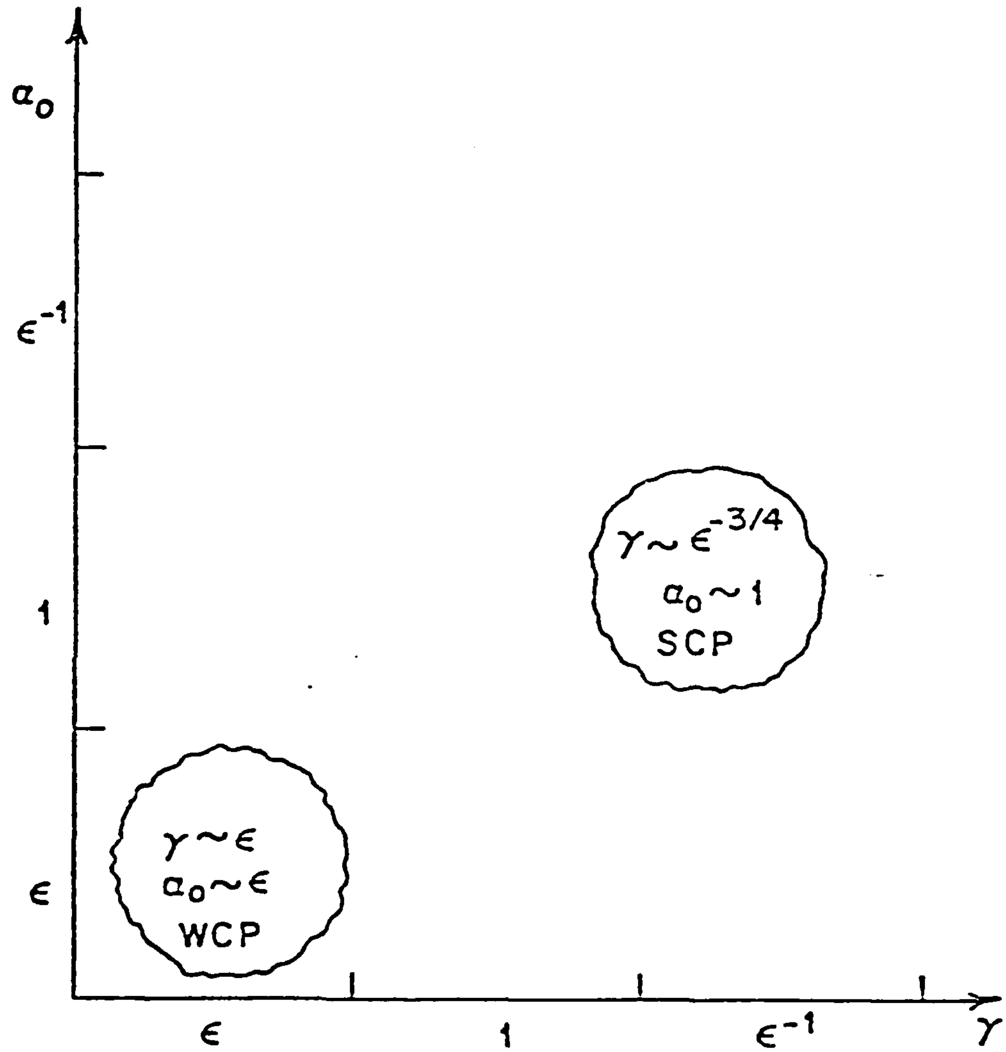

Nondimensionalized BBGKY hierarchy
Mayer Expansion (7)
Correlation Ordering(γ) and Expansion (ϵ) (10)
Interaction-Force Expansion (6)
Weak Coupling $\gamma \ll 1$
Strong Coupling $\gamma \gg 1$
BY_1 & BY_2 for $h^{(0)}$
BY_1 & BY_2 for $h^{(0)}$
Vlasov Eq. Debye-Hückle Correlation
Decaying Oscillation for $y^2 > \frac{\kappa}{2}$

FIG. 2



a_0
ϵ^{-1}
1
ϵ
$\gamma \sim \epsilon^{-3/4}$
$a_0 \sim 1$
SCP
$\gamma \sim \epsilon$
$a_0 \sim \epsilon$
WCP
ϵ
1
ϵ^{-1}
γ

FIG. 3

END

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